(Affiliated to University of Mumbai)	***End Semester Examination (R-24) SH 2025 Answer Key with marking scheme***
Branch: CE/AIDS/AIML/CSEIOT	Course: Applied Mathematics III
Year/ Semester: SE / III	Course code: CEC301/AIDSC301/AIMLC301/CSEC301
Time: 03 hours	Marks: 80

		Marks
Q. 1	Attempt any FOUR. (All questions carry equal marks)	
A.	→ $\frac{1}{t}$ is a property. $f(t) = \sin t \cdot \sin 5t = \frac{1}{2}[\cos 4t - \cos 6t]$ $L\{\sin t \cdot \sin 5t\} = \frac{1}{2}\left[\frac{s}{s^2+16} - \frac{s}{s^2+36}\right]$ $L\left\{\frac{\sin t \cdot \sin 5t}{t}\right\} = \frac{1}{2}\int_s^{\infty}\frac{1}{2}\left[\frac{2s}{s^2+16} - \frac{2s}{s^2+36}\right]ds$ $= \frac{1}{2}\cdot\frac{1}{2}\left[\log\left(\frac{s^2+16}{s^2+36}\right)\right]_s^{\infty}$ $= \frac{1}{4}\left[(0) - \log\left(\frac{s^2+16}{s^2+36}\right)\right]$ $= \frac{1}{4}\log\left(\frac{s^2+36}{s^2+16}\right)$ #	5
B.	→ $(-\pi,\pi) \equiv (-l,l) \Rightarrow l=\pi$ $f(x)=x^2$, $f(-x) = (-x)^2 = x^2 = f(x) \Rightarrow$ even function $\Rightarrow b_n = 0$ $\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cdot \cos nx + 0}$ —(1) $a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx = \frac{2}{l}\int_0^l f(x)\,dx$ $= \frac{2}{\pi}\int_0^{\pi} x^2\,dx$ $= \frac{2}{\pi}\left[\frac{x^3}{3}\right]_0^{\pi}$ $\boxed{a_0 = \frac{2\pi^2}{3}}$ $a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cdot\cos\left(\frac{n\pi x}{l}\right)dx = \frac{2}{\pi}\int_0^{\pi} x^2 \cdot \cos nx\,dx$ $= \frac{2}{\pi}\left[x^2\left(\frac{\sin nx}{n}\right) - 2x\left(\frac{-\cos nx}{n^2}\right) + 2\left(-\frac{\sin nx}{n^3}\right)\right]_0^{\pi}$ $a_n = \frac{2}{\pi}\left[\frac{2\pi \cdot \cos n\pi}{n^2}\right]$ $\therefore \boxed{a_n = \frac{4(-1)^n}{n^2}}$ (1) $\Rightarrow \boxed{x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cdot\cos nx}$	5
C.		5

	$\rightarrow M_0(t) = \sum P_i e^{tx_i} = \frac{1}{3}e^{-2t} + \frac{1}{2}e^{3t} + \frac{1}{6}e^{t}$ = MGF Momen Mean → centr $\mu_r' = \frac{d^r}{dt^r}[M_0(t)]_{t=0}$ $\mu_1' = \frac{d}{dt}[M_0(t)]_{t=0} = \frac{d}{dt}\left[\frac{1}{3}e^{-2t} + \frac{1}{2}e^{3t} + \frac{1}{6}e^{t}\right]_{t=0}$ $= \left[\frac{1}{3}(-2)e^{-2t} + \frac{3}{2}e^{3t} + \frac{1}{6}e^{t}\right]_{t=0}$ $\sum x_i p_i$ = mean $= -\frac{2}{3} + \frac{3}{2} + \frac{1}{6}$ $= \frac{-4+9+1}{6} = \boxed{1}$ = mean $\mu_2' = \frac{d^2}{dt^2}\left[\frac{1}{3}e^{-2t} + \frac{1}{2}e^{3t} + \frac{1}{6}e^{t}\right]_{t=0}$ $= \frac{d}{dt}\left[-\frac{2}{3}e^{-2t} + \frac{3}{2}e^{3t} + \frac{1}{6}e^{t}\right]_{t=0}$ $= \frac{4}{3}e^{-2t} + \frac{9}{2}e^{3t} + \frac{e^{t}}{6}\Big]_{t=0}$ $= \frac{4}{3} + \frac{9}{2} + \frac{1}{6}$ $= \frac{8+27+1}{6} = \frac{36}{6} = \boxed{6} = \mu_2'$	
D.	**Example 08: Find $L^{-1}\left[\frac{s^2}{(s^2+1)(s^2+4)}\right]$** $\rightarrow \frac{s^2}{(s^2+1)(s^2+4)}$, $s^2 = x$ $\frac{x}{(x+1)(x+4)} = \frac{A}{x+1} + \frac{B}{x+4}$ $x = A(x+4) + B(x+1)$ $x=-1$; $-1 = 3A \Rightarrow A = -1/3$ $x=-4$; $-4 = -3B \Rightarrow B = 4/3$ $f(s) = -\frac{1}{3}\frac{1}{s^2+1} + \frac{4}{3}\cdot\frac{1}{s^2+4}$ (2×2) Taking ILT $f(t) = -\frac{1}{3}\sin t + \frac{2}{3}\cdot\sin 2t$	5
E.	(table below)	5

x	y	d_x	d_y	$d_x d_y$	d_x^2	d_y^2
100	30	-200	-20	4000	40000	400
200	40	-100	-10	1000	10000	100
300	50	0	0	0	0	0
400	60	100	10	1000	10000	100
500	70	200	20	4000	40000	400
N = 5		$\sum d_x = 0$	$\sum d_y = 0$	$\sum d_x d_y = 10{,}000$	$\sum d_x^2 = 100000$	$\sum d_y^2 = 1000$

$$\frac{10000}{\sqrt{100000 \times 1000}} = \frac{10000}{10000} = \boxed{1}$$

F. (5)

Sol. : We have $v = e^{-x}(y\sin y + x\cos y)$

$$\therefore \quad v_y = \psi_1(x,y) = e^{-x}(\sin y + y\cos y - x\sin y)$$

$$v_x = \psi_2(x,y) = -e^{-x}(y\sin y + x\cos y) + e^{-x}(\cos y)$$

$$\therefore \quad v_x = \psi_2(x,y) = e^{-x}(\cos y - y\sin y - x\cos y)$$

$$\therefore \quad \psi_1(z,0) = 0, \quad \psi_2(z,0) = e^{-z}(1-z)$$

By Milne-Thompson Method

$$f'(z) = \psi_1(z,0) + i\,\psi_2(z,0)$$

$$\therefore \quad f(z) = \int \psi_1(z,0)\,dz + i\int \psi_2(z,0)\,dz = i\int e^{-z}(1-z)\,dz$$

$$= i\left[(1-z)(-e^{-z}) - \int -e^{-z}(-1)\,dz\right] = i\,[(1-z)(-e^{-z}) + e^{-z}]$$

$$= i\,e^{-z}z + c.$$

Q.2 Attempt any FOUR. (All questions carry equal marks)

A. (10)

Example 05*: *Find $L^{-1}\left[\frac{1}{(s^2+1)^2}\right]$

$\phi_1(s) = \frac{1}{s^2+1} \rightarrow f_1(t) = \sin t$

$\phi_2(s) = \frac{1}{s^2+1} \rightarrow f_2(t) = \sin t$

$$L^{-1}\{\phi(s)\} = \int_0^t \sin u \cdot \sin(t-u)\,du$$

$$= \frac{1}{2}\int_0^t [\cos(u-t+u) - \cos(u+t-u)]\,du$$

$$= \frac{1}{2}\int_0^t [\cos(2u-t) - \cos t]\,du$$

$$= \frac{1}{2}\left[\frac{\sin(2u-t)}{2} - u\cos t\right]_0^t$$

$$= \frac{1}{2}\left[\left[\frac{\sin t}{2} - t\cos t\right] - \left[-\frac{\sin t}{2} - 0\right]\right]$$

$$= \frac{1}{2}\left[\sin t - t\cos t\right]$$

Ans.

B.

→ $(0,2\pi) \equiv (0,2l) \Rightarrow l=\pi$, $f(x) = \frac{a_0}{2} + \sum a_n \cos nx + \sum b_n \sin nx$ — ①

$a_0 = \frac{1}{\pi}\left[\int_0^{\pi} \sin x\,dx + \int_{\pi}^{2\pi} 0\,dx\right] = \frac{1}{\pi}\left[-\cos x\right]_0^{\pi} = -\frac{1}{\pi}\left[-1-1\right] = \boxed{\frac{2}{\pi}}$

$a_n = \frac{1}{\pi}\left[\int_0^{\pi} \sin x \cdot \cos nx\,dx\right] = \frac{1}{\pi}\int_0^{\pi} \frac{1}{2}\left[\sin(1+n)x + \sin(1-n)x\right]dx$

② — $\boxed{a_n = \frac{1}{2\pi}\int_0^{\pi} \left[\sin(n+1)x - \sin(n-1)x\right]dx}$

$\cos(n\pm1)\pi = -(-1)^n = (-1)^{n+1}$

$= \frac{1}{2\pi}\left[-\frac{\cos(n+1)x}{n+1} + \frac{\cos(n-1)x}{n-1}\right]_0^{\pi}$

$= \frac{1}{2\pi}\left[\left[\frac{(-1)^n}{n+1} - \frac{(-1)^n}{n-1}\right] - \left[\frac{-1}{n+1} + \frac{1}{n-1}\right]\right]$

$= \frac{1}{2\pi}\left[(-1)^n\left(\frac{1}{n+1} - \frac{1}{n-1}\right) + 1\left(\frac{1}{n+1} - \frac{1}{n-1}\right)\right]$

$= \frac{1}{2\pi}\left[\left[(-1)^n+1\right]\left[\frac{n-1-n-1}{n^2-1}\right]\right]$

$= \frac{1}{2\pi}\left[(-1)^n+1\right]\left[\frac{-2}{n^2-1}\right]$

$\sum^{\infty}$ ①

$\boxed{a_n = \frac{-1}{\pi(n^2-1)}\left[(-1)^n+1\right]}$

$= \begin{cases} 0 & : n \text{ is odd} \\ \frac{-2}{\pi(n^2-1)} & : n \text{ is even} \end{cases}$

But method fails for $n=1$

put $n=1$ in ②

$a_1 = \frac{1}{2\pi}\int_0^{\pi} (\sin 2x)\,dx$

$= \frac{1}{2\pi}\left[\frac{-\cos 2x}{2}\right]_0^{\pi}$

$= -\frac{1}{4\pi}\left[1-1\right]$

$\boxed{a_1 = 0}$

→ $b_n = \frac{1}{\pi}\left[\int_0^{\pi} \sin x \cdot \sin nx \cdot dx + 0\right]$

$= \frac{1}{2\pi}\int_0^{\pi} \left[\cos(1-n)x - \cos(1+n)x\right]dx$

$= \frac{1}{2\pi}\int_0^{\pi} \left[\cos(n-1)x - \cos(n+1)x\right]dx$ — ③

$b_n = \frac{1}{2\pi}\left[\frac{\sin(n-1)x}{n-1} - \frac{\sin(n+1)x}{n+1}\right]_0^{\pi}$

$= 0$ #

Method fails for $n=1$, put $n=1$ in

$b_1 = \frac{1}{2\pi}\int_0^{\pi} (1 - \cos 2x)\,dx$

$= \frac{1}{2\pi}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi} = \frac{1}{2\pi}\left[(\pi-0)-(0-0)\right]$

$= \boxed{\frac{1}{2}} \neq 0$

① ⇒

$f(x) = \frac{a_0}{2} + a_1\cos x + \sum_{n=2}^{\infty} a_n \cos nx + b_1 \sin x + \sum_{n=2}^{\infty} b_n \sin nx$

$= \frac{1}{2}\cdot\frac{2}{\pi} + 0 + \sum_{n \text{ is even}}^{\infty} \frac{-2}{\pi(n^2-1)}\cdot\cos nx + \frac{1}{2}\sin x + 0$

$\boxed{f(x) = \frac{1}{\pi} - \frac{2}{\pi}\sum_{n \text{ is even}} \frac{\cos nx}{n^2-1} + \frac{1}{2}\sin x}$

$\boxed{x=0}$

$0 = \frac{1}{\pi} - \frac{2}{\pi}\sum_{n \text{ even}} \frac{1}{n^2-1} + 0$

$0 = \frac{1}{\pi} - \frac{2}{\pi}\left[\frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \cdots\right]$

$\left(+\frac{1}{\pi}\right)\left(\frac{\pi}{2}\right) = \frac{1}{1\cdot3} + \frac{1}{3\cdot5} + \frac{1}{5\cdot7} + \cdots$

$\boxed{\frac{1}{2} = \frac{1}{1\cdot3} + \frac{1}{3\cdot5} + \frac{1}{5\cdot7} + \cdots}$

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C.	$u-v=U=\dfrac{\cos x+\sin x-e^{-y}}{2\cos x-e^{y}-e^{-y}}$ $U_x=\dfrac{(2\cos x-e^{y}-e^{-y})(-\sin x+\cos x)-(\cos x+\sin x-e^{-y})(-2\sin x)}{(2\cos x-e^{y}-e^{-y})^2}\xrightarrow[y=0]{x=z}$ $\phi_1=\dfrac{(2\cos z-2)(-\sin z+\cos z)-(\cos z+\sin z-1)(-2\sin z)}{(2\cos z-2)^2}$ $\phi_1=\dfrac{-2\sin z\cos z+2\cos^2 z+2\sin z-2\cos z+2\sin z\cos z+2\sin^2 z-2\sin z}{(2\cos z-2)^2}$ $\phi_1=\dfrac{2-2\cos z}{(2-2\cos z)^2}=\dfrac{1}{2(1-\cos z)}=\dfrac{1}{2\cdot 2\sin^2(z/2)}$ $\boxed{\phi_1=\frac{1}{4}\operatorname{cosec}^2(z/2)}$ $U_y=\dfrac{(2\cos x-e^{y}-e^{-y})(e^{-y})-(\cos x+\sin x-e^{-y})(-e^{y}+e^{-y})}{(2\cos x-e^{y}-e^{-y})^2}\xrightarrow[y=0]{x=z}$ $\phi_2=\dfrac{(2\cos z-2)-0}{(2\cos z-2)^2}=\dfrac{1}{-2(1-\cos z)}=\dfrac{1}{-2\cdot 2\sin^2(z/2)}=-\frac{1}{4}\operatorname{cosec}^2(z/2)$ By M-T method $(1+i)f'(z)=\phi_1-i\phi_2$ $=\frac{1}{4}\operatorname{cosec}^2(z/2)+\frac{i}{4}\operatorname{cosec}^2(z/2)$ $(1+i)f'(z)=\frac{(1+i)}{4}\operatorname{cosec}^2(z/2)$ $f'(z)=\frac{1}{4}\cdot\operatorname{cosec}^2(z/2)$ on integration $f(z)=\frac{1}{4}\left\{\dfrac{-\cot(z/2)}{1/2}\right\}+c$ $\boxed{f(z)=-\frac{1}{2}\cot(z/2)+c}$ put $z=\pi/2$ $f(\pi/2)=-\frac{1}{2}\cot(\frac{\pi}{4})+c$ $0=-\frac{1}{2}+c$ $\boxed{c=\frac{1}{2}}$ $\therefore f(z)=-\frac{1}{2}\cot(\frac{z}{2})+\frac{1}{2}$ $\boxed{f(z)=\frac{1}{2}(1-\cot\frac{z}{2})}$	10
D.	**Example 11: Show that** $\int_0^\infty e^{-\sqrt{2}\,t}\,\dfrac{\sin t\,\sinh t}{t}\,dt=\dfrac{\pi}{8}$ $\to s=\sqrt{2}$, $\sinh t\cdot\sin t=\frac{1}{2}(e^{t}-e^{-t})\sin t$ $=\frac{1}{2}[e^{t}\cdot\sin t-e^{-t}\sin t]$ $L\{\sinh t\cdot\sin t\}=\frac{1}{2}\left[\dfrac{1}{(s-1)^2+1}-\dfrac{1}{(s+1)^2+1}\right]$ $L\left\{\dfrac{\sinh t\cdot\sin t}{t}\right\}=\frac{1}{2}\int_s^\infty\left[\dfrac{1}{(s-1)^2+1^2}-\dfrac{1}{(s+1)^2+1^2}\right]ds$ $=\frac{1}{2}\left[\tan^{-1}(\frac{s-1}{1})-\tan^{-1}(s+1)\right]_s^\infty$ $(\frac{\pi}{2}-\frac{\pi}{2})$ $=0-\frac{1}{2}\left[\tan^{-1}(s-1)-\tan^{-1}(s+1)\right]$ $\tan^{-1}A-\tan^{-1}B=\tan^{-1}\left[\dfrac{A-B}{1+AB}\right]$ $=\frac{1}{2}\left[\tan^{-1}(s+1)-\tan^{-1}(s-1)\right]$ $=\frac{1}{2}\tan^{-1}\left[\dfrac{s+1-s+1}{1+(s+1)(s-1)}\right]$ $=\frac{1}{2}\tan^{-1}\left[\dfrac{2}{1+s^2-1}\right]$ $=\frac{1}{2}\tan^{-1}(\frac{2}{s^2})$, $\boxed{\text{put } s=\sqrt{2}}$ $I=\frac{1}{2}\tan^{-1}(\frac{2}{2})$ $=\frac{1}{2}\frac{\pi}{4}=\frac{\pi}{8}$ #	10

E. (10)

Sr. No.	x	x^2	y	y^2	$\sum xy$
1	5	25	11		
2	6	36	14		
3	7		14		
4	8		15		
5	9		12		
6	10		17		
7	11		16		
Sum	56	476	99	1427	811

1) y on x

$$b_{yx} = \frac{811 - \frac{56\times 99}{7}}{476 - \frac{(56)^2}{7}} = 0.679$$

$$\bar{x} = \frac{56}{7} = 8, \quad \bar{y} = \frac{99}{7} = 14.14$$

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - 14.14 = 0.679(x - 8)$$

$$\boxed{y = 0.679x + 8.71}$$

$$y = a + bx$$

$$b_{xy} = \frac{811 - \frac{56\times 99}{7}}{1427 - \frac{99^2}{7}} = 0.7074$$

$$r = \sqrt{b_{xy} \times b_{yx}} = \sqrt{0.7074 \times 0.6786} = \boxed{0.69}$$

$$\boxed{x = 0.7074y - 2.0026}$$

F. (10)

We know that $\int_0^1 k(x - x^2)\,dx = 1$

$$k\left(\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 1$$

$$k\left[\frac{1}{2} - \frac{1}{3}\right] = 1$$

$$\boxed{k = 6}, \quad f(x) = 6(x - x^2)$$

$$\text{Mean} = \int x f(x)\,dx = \int_0^1 6(x^2 - x^3)\,dx$$

$$= 6\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$$

$$= 6\left[\frac{1}{3} - \frac{1}{4}\right] = \boxed{\frac{1}{2}} = \mu_1'$$

$$\mu_2' = \int x^2 f(x)\,dx = \int_0^1 6(x^3 - x^4)\,dx$$

$$= 6\left[\frac{x^4}{4} - \frac{x^5}{5}\right]_0^1$$

$$= 6\left[\frac{1}{4} - \frac{1}{5}\right] = 6\left[\frac{1}{20}\right] = \boxed{3/10}$$

$$\therefore \text{Variance} = \mu_2' - \mu_1'^2$$

$$= \frac{3}{10} - \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{10} - \frac{1}{4} = \frac{2}{40} = \boxed{\frac{1}{20}}$$

Q.3 Attempt any FOUR. (All questions carry equal marks)

A. (5)

x	y	R_x	R_y	$d_i = R_x - R_y$	d_i^2
65	67	8	7	1	1
66	68	7	5.5	1.5	2.25
67	64	5.5	8	-2.5	6.25
67	68	5.5	5.5	0	0
68	72	4	1	3	9
69	70	3	2.5	0.5	0.25
71	69	2	4	-2	4
73	70	1	2.5	-1.5	2.25
$n=8$				$\sum d_i^2 = 25$	

factor = 1.5

$$\therefore R = 1 - \left[\frac{6[25 + 1.5]}{512 - 8}\right]$$

$$= 0.6845 \checkmark$$

B. (5)

$P_1 = p(\text{having cancer}) = \frac{2000}{100000} = \frac{2}{100} = 0.02$

$P_2 = p(\text{having no. cancer}) = 1 - 0.02 = 0.98$

$P_1' = p(\text{person having cancer}) = 0.95$

$P_2' = p(\text{person not having cancer}) = 0.05$

$$\text{Expected probability} = \frac{P_1 P_1'}{P_1 P_1' + P_2 P_2'} = \frac{0.02 \times 0.95}{(0.02 \times 0.95) + (0.98 \times 0.05)}$$

$$= \underline{0.279}$$

$$= \frac{19}{68}$$

C. (5)

Sol. : Let $f(x) = \sum b_n \sin nx$

$$\therefore \quad b_n = \frac{2}{\pi}\int_0^{\pi} f(x) \sin nx\, dx = \frac{2}{\pi}\int_0^{\pi} x(\pi - x) \sin nx\, dx$$

$$= \frac{2}{\pi}\left[x(\pi - x)\left(-\frac{\cos nx}{n}\right) - (\pi - 2x)\left(-\frac{\sin nx}{n^2}\right) + (-2)\left(\frac{\cos nx}{n^3}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\left\{0 - 0 - \frac{2}{n^3}\cos(n\pi)\right\} - \left\{0 - 0 - \frac{2}{n^3}\right\}\right]$$ [By § 5, page 3-

$$= \frac{4}{\pi}\left[\frac{1 - \cos n\pi}{n^3}\right] = \frac{4}{\pi n^3}\left[1 - (-1)^n\right]$$ [$\because$ $\cos n\pi = (-$

$$\therefore \quad b_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{8}{\pi n^3} & \text{if } n \text{ is odd} \end{cases}$$

$$\therefore \quad x(\pi - x) = \frac{8}{\pi}\left[\frac{1}{1^3}\sin \pi x + \frac{1}{3^3}\sin 3\pi x + \frac{1}{5^3}\sin 5x + \ldots\right]$$

D. (5)

$\rightarrow$ Let $u = e^{-x}\cos y + xy$

$u_x = -e^{-x}\cdot\cos y + y \xrightarrow[y=0]{x=z} \phi_1 = -e^{-z}$

$u_y = -e^{-x}\cdot\sin y + x \xrightarrow[y=0]{x=z} \phi_2 = z$

By M.T method

$f'(z) = \phi_1 - i\phi_2$

$= -e^{-z} - iz$

on int^n

$f(z) = e^{-z} - i\frac{z^2}{2} + c$

$f(z) = e^{-(x+iy)} - \frac{i}{2}(x+iy)^2 + c$

$= e^{-x}[\cos y - i\sin y] - \frac{i}{2}(x^2 + 2ixy - y^2) + c$

$= e^{-x}\cos y - ie^{-x}\sin y - \frac{i}{2}x^2 + xy + \frac{i}{2}y^2 + c$

$= \underline{(e^{-x}\cos y + xy)} + i\left(-e^{-x}\sin y - \frac{x^2}{2} + \frac{y^2}{2}\right) + c$

$\therefore$ $v = -e^{-x}\sin y - \left(\frac{x^2 - y^2}{2}\right) + c$

$\therefore$ O.T is $\boxed{-e^{-x}\sin y - \left(\frac{x^2 - y^2}{2}\right) = \alpha}$ ✓

<table>
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<td>E.</td>
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$\rightarrow f(u) = \sin 3u$

$L\{\sin 3u\} = \frac{3}{s^2+9}$

$L\{u \cdot \sin 3u\} = -\frac{d}{ds}\left[\frac{3}{s^2+9}\right]$

$= -3\left[-\frac{2s}{(s^2+9)^2}\right]$

$= \frac{6s}{(s^2+9)^2}$

$L\{\int_0^t u \sin 3u \, du\} = \frac{1}{\cancel{s}} \frac{6\cancel{s}}{(s^2+9)^2} = \frac{6}{(s^2+9)^2}$

$L\{e^{-4t}\int_0^t u \cdot \sin 3u \, du\} = \frac{6}{[(s+4)^2+9]^2}$

$= \frac{6}{[s^2+8s+25]^2}$

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<td>F.</td>
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Sol. : Taking Laplace transforms of both sides,

$L(y') + 3L(y) = L(2) + L(e^{-t})$

$s\bar{y} - y(0) + 3\bar{y} = 2\frac{1}{s} + \frac{1}{s+1}$. But $y(0) = 1$

$\therefore \ (s+3)\,\bar{y} = \frac{2}{s} + \frac{1}{s+1} + 1 = \frac{s^2+4s+2}{s(s+1)}$

$\therefore \ \bar{y} = \frac{s^2+4s+2}{s(s+1)(s+3)}$

$\therefore$ By partial fractions $\bar{y} = \frac{2}{3}\cdot\frac{1}{s} + \frac{1}{2}\cdot\frac{1}{s+1} - \frac{1}{6}\cdot\frac{1}{s+3}$

Taking inverse Laplace transforms, $y = \frac{2}{3} + \frac{1}{2}\cdot e^{-t} - \frac{1}{6}e^{-3t}$

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